Spectral theory of simple resets

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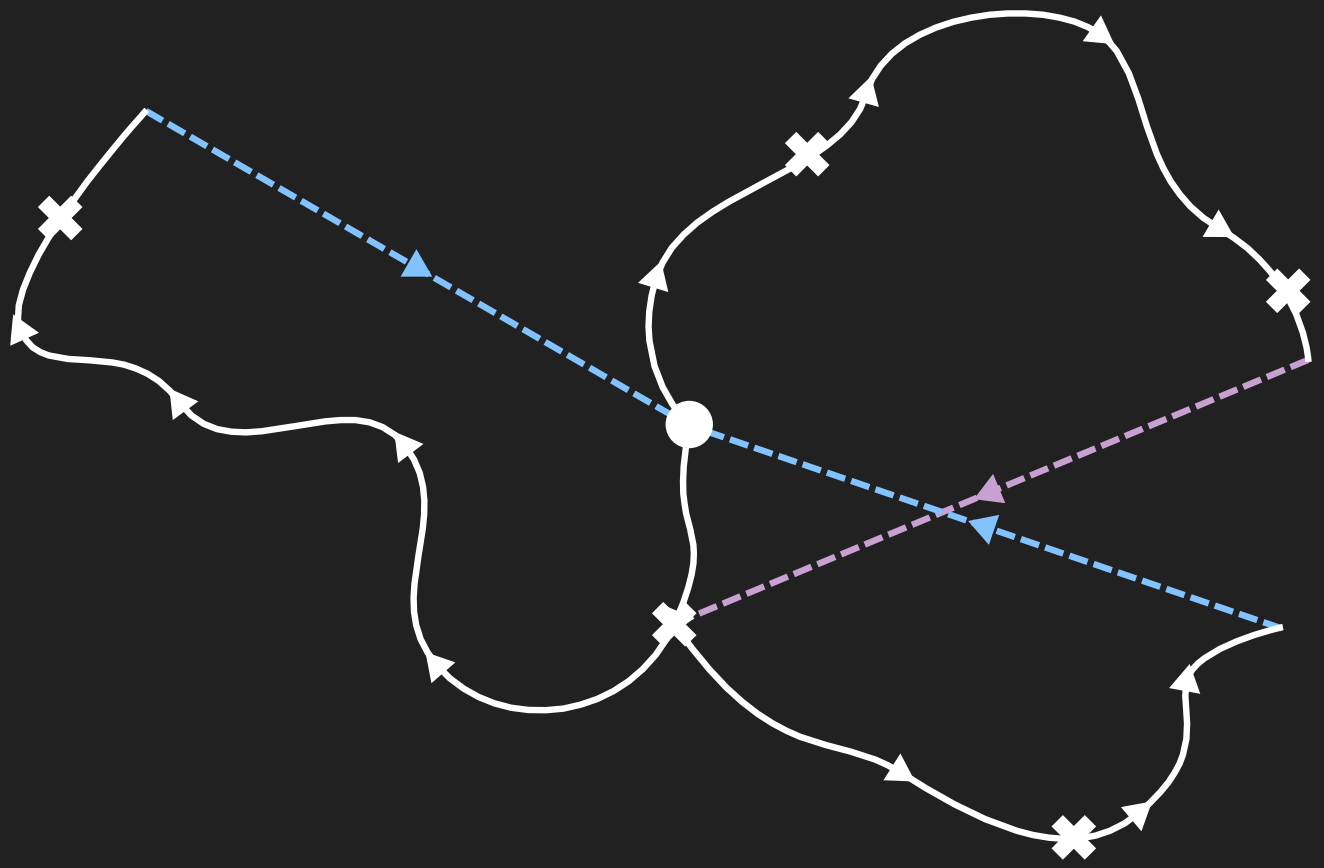
Spectral properties of simple classical and quantum reset processes,
PRE 98, 022129 (2018)

Resets in nature

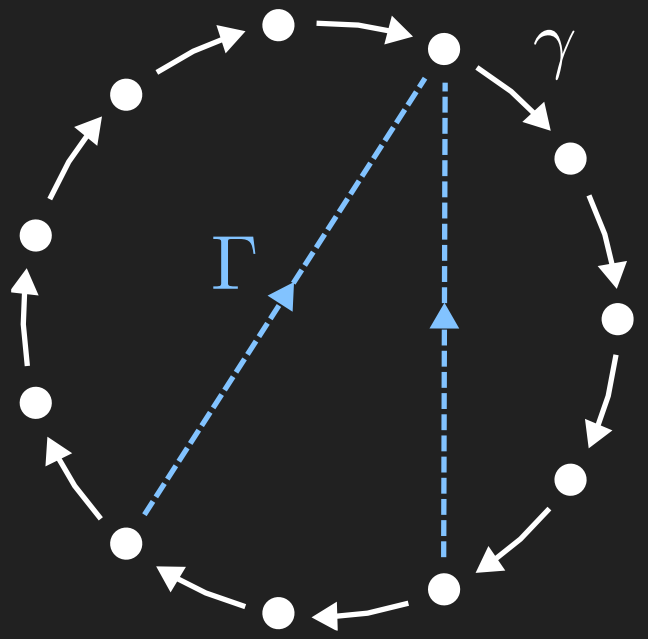


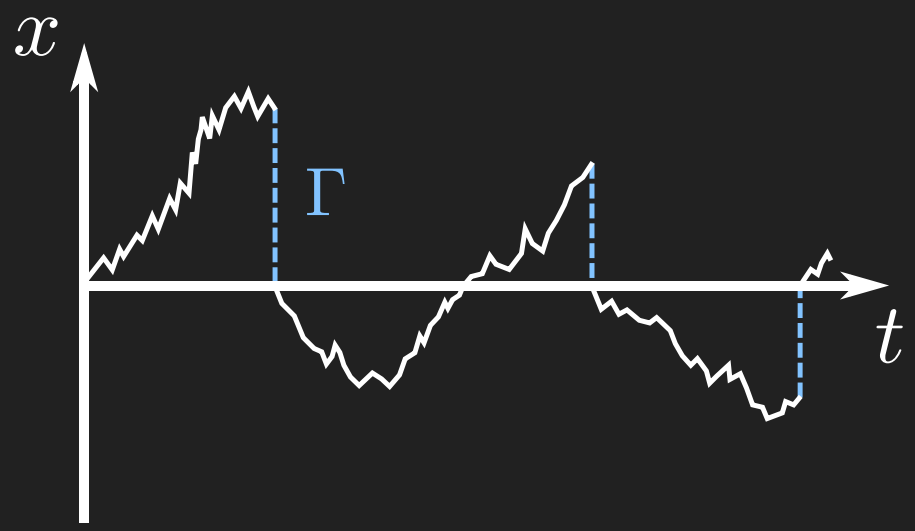
- Foraging
- Car keys

Outline

- Spectra of master operators
- Resets, spectral results
- Quantum systems
- Metastability

Jumps and diffusion





- Cyclic hopping
- Brownian motion

Master operators

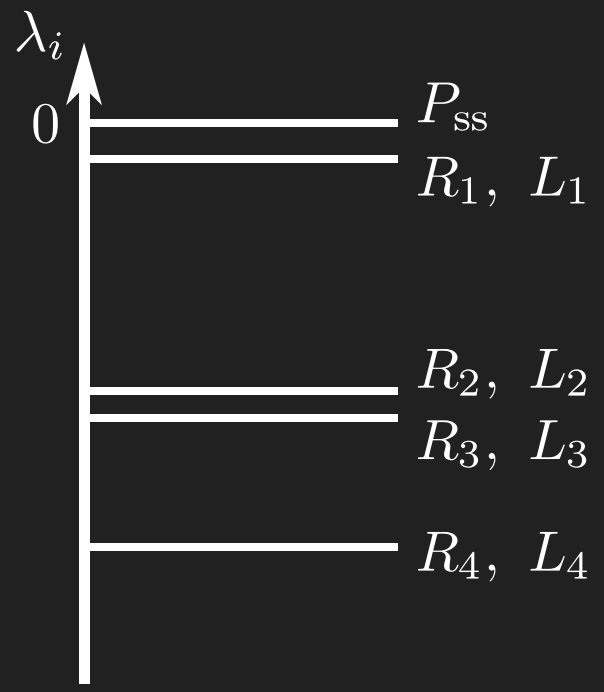
- Rate matrix

$$\frac{dP}{dt} = \mathcal{L}P$$

- Linear $\longrightarrow$ Spectrum

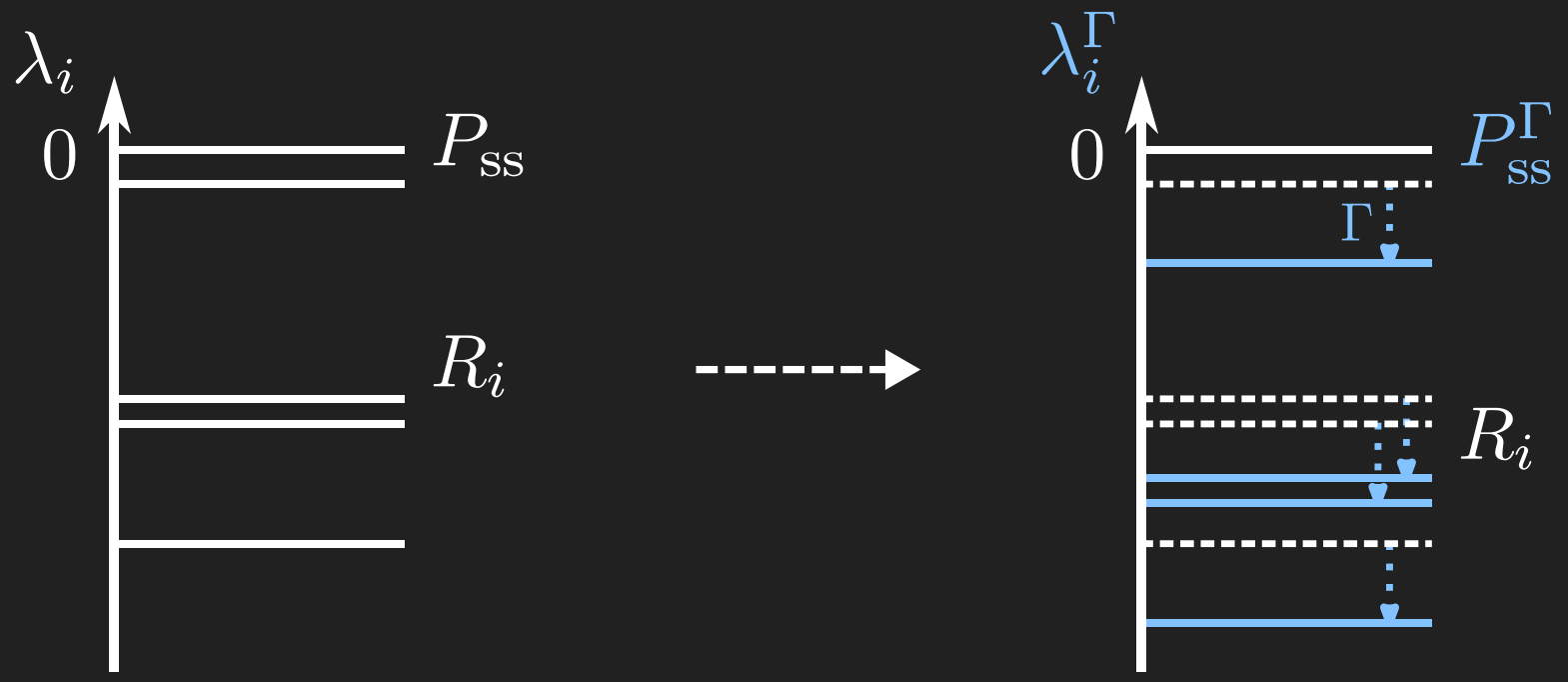
$$\mathcal{L}R_i = \lambda_i R_i \qquad \mathcal{L}^\dagger L_i = \lambda_i^* L_i$$

- Fokker-Planck



Results 1: Spectral shift

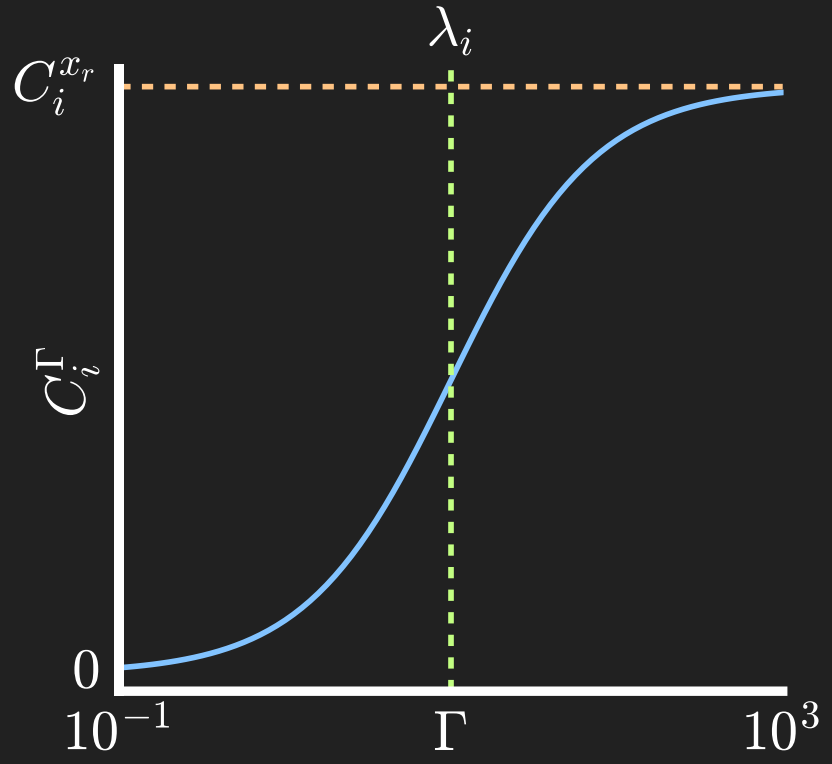
$$\mathcal{L}_\Gamma = \mathcal{L} + \Gamma(\text{all } x \to x_r) - \Gamma I$$



Results 2: stationary state

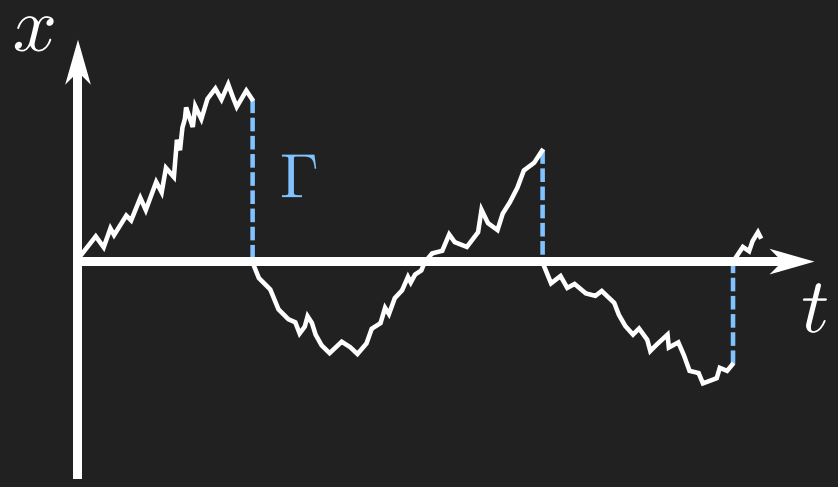
$$L_i \to L_i - c_i^{\Gamma} I$$

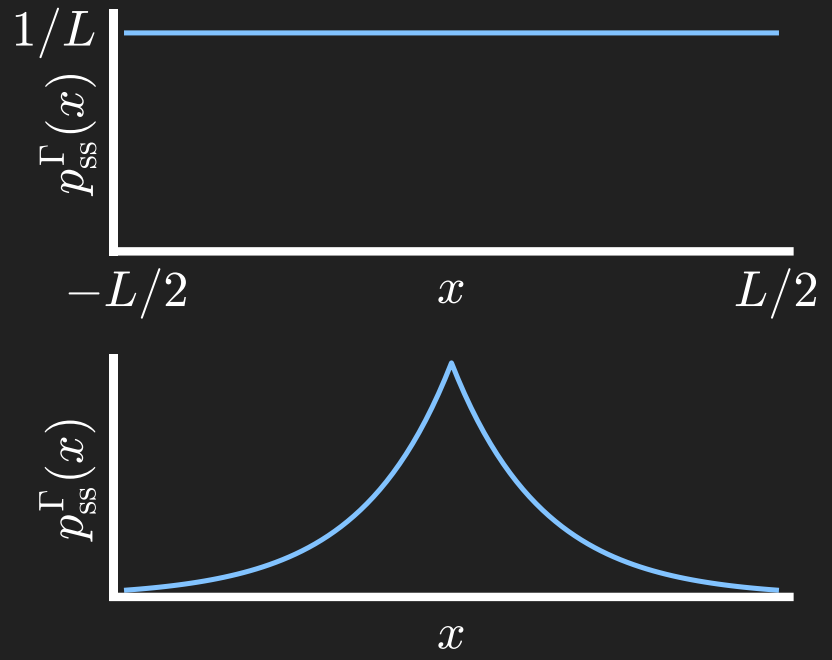
$$P_{\mathrm{ss}}^{\Gamma} = P_{\mathrm{ss}} + \sum_i c_i^{\Gamma} R_i$$



- Non-equilibrium
- Induces relaxation

Example: Brownian motion





- Exponential decay

M. R. Evans et al., Phys. Rev. Lett. **106**, 160601

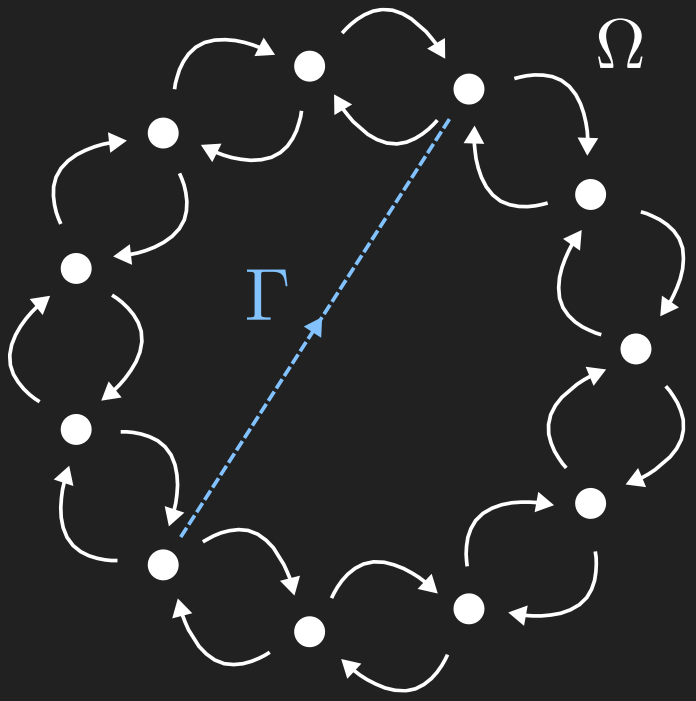
Quantum systems

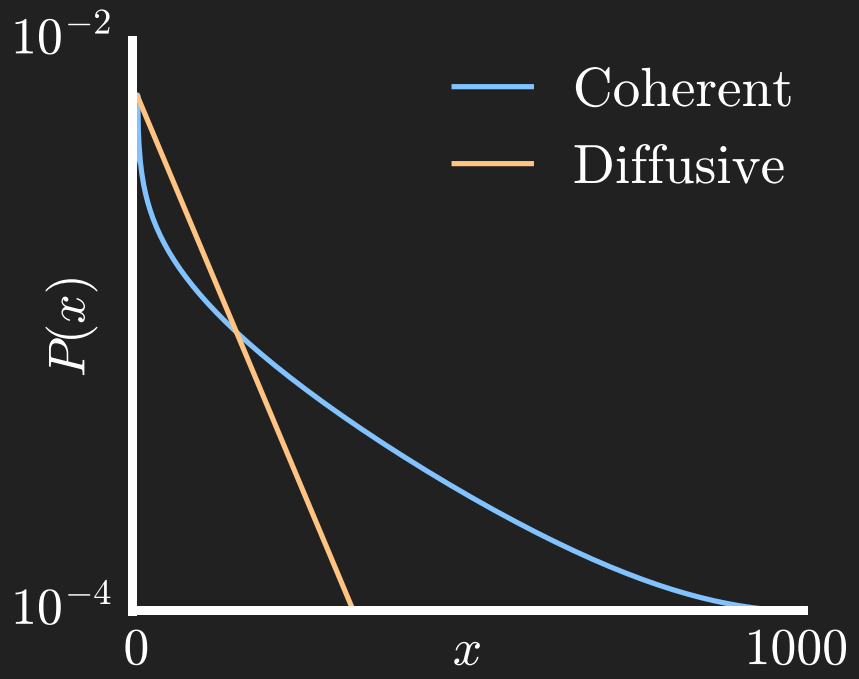
$$\frac{d\rho}{dt} = \mathcal{L}(\rho)$$

- E.g. closed systems: $\mathcal{L}(\rho) = -i[H, \rho]$
- Similar spectrum to classical case

$$\mathcal{L}_\Gamma = \mathcal{L} + \Gamma(\text{all } \rho \to |\psi_r\rangle\langle\psi_r|) - \Gamma I$$

Example: Coherent hopping

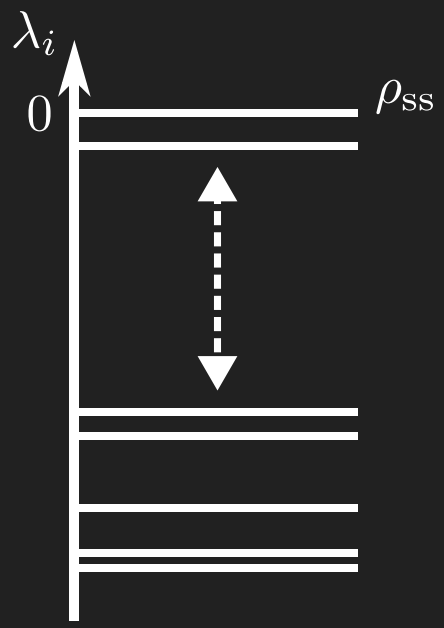


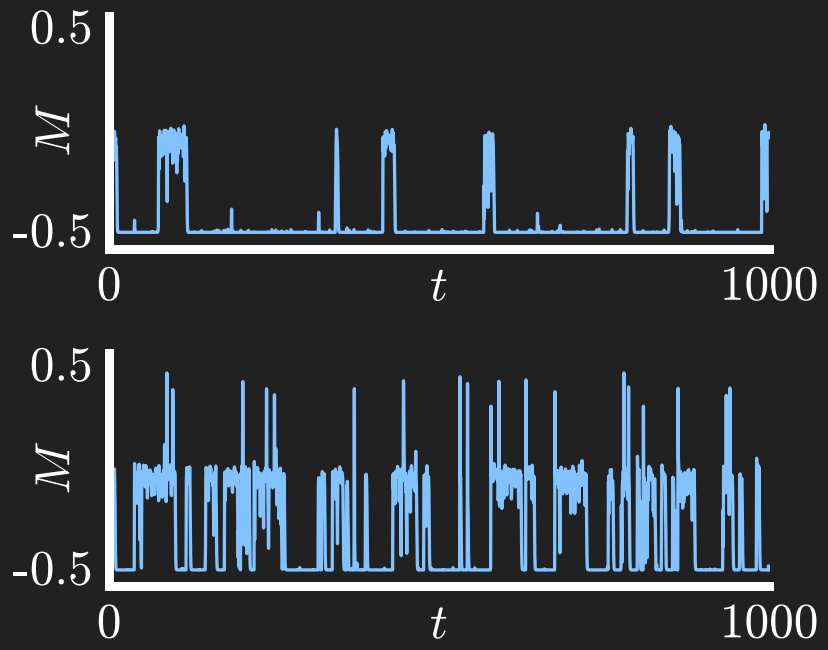


- Ballistic motion ⟶ Slower decay

Metastability

- Spectral gap ⟶ long-lived states





- Weak resets ⟶ bias metastability

DCR et al., Phys. Rev. E **94**, 052132

Summary

- Simple resets cause simple changes
  - Spectral shift
  - Remnants of dynamical modes
- Affects classical and quantum processes equivalently
- May bias metastable states

Outlook

- Little known about resets in quantum systems
- Spectra of large deviations with simple resets